Constrained Model Reference Adaptive Control with Applications to Tail-Sitter UAV Control Systems Design

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IEEE Conference on Control Technology and Applications

August 15, 2023

Tailsitter Unmanned Aerial Vehicles

- Tailsitter popularity rising due to potential applications:
  - ISR;
  - Payload transport;
  - Search and rescue.
- Best of both worlds?
  - Maneuverability of multirotors;
  - Efficiency and endurance of fixed wings.
- Typically captured by two modes:
  - Multirotor mode or VTOL mode;
  - Fixed-wing mode;
  - Sometimes a third mode: transition mode.
- **Large flight envelope**
  - AoA will pass stall during transition

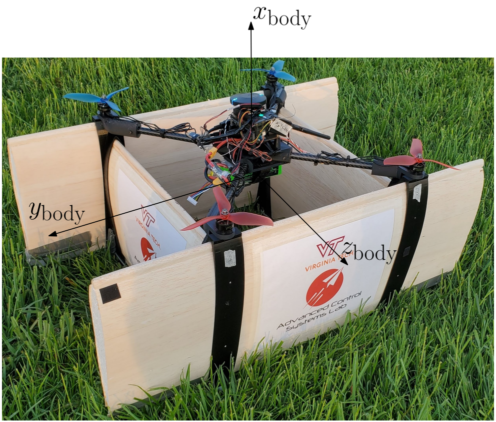
Literature Review on Tailsitter Control

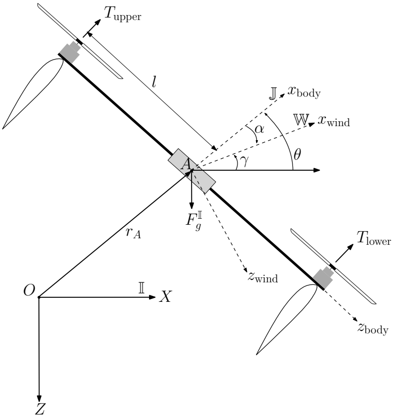
Control methods used on tailsitter UAVs include:

- Open loop transition control;
- PID, Gain-scheduled LQR controllers;
- Adaptive backstepping, $\mathcal{L}_1$ adaptive control;
- Nonlinear dynamic inversion.

Classical MRAC has been applied to a tailsitter UAV's pitch dynamics or linearized longitudinal dynamics.

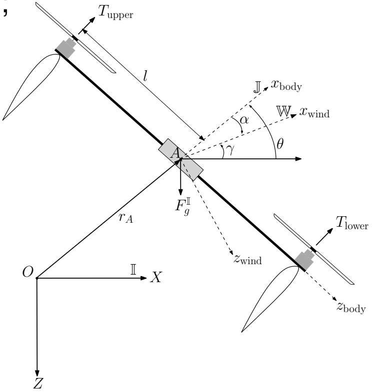
Modeling Assumptions





Modeling Assumptions

- *Inertial* reference frame $\mathbb{I} \triangleq \{O; X, Y, Z\}$;
- *Body* reference frame $\mathbb{J}(\cdot) \triangleq \{A(\cdot); x_{\text{body}}(t), y_{\text{body}}(t), z_{\text{body}}(t)\}$;
- *Wind* reference frame $\mathbb{W}(\cdot) \triangleq \{A(\cdot); x_{\text{wind}}(t), y_{\text{wind}}(t), z_{\text{wind}}(t)\}$;
- Symmetric vehicle, NACA 0012 airfoil,
- Longitudinal motion only, $y_{\text{body}}(t) \equiv Y$, $t \geq t_0$, $x_{\text{wind}}^{\mathrm{T}}(t) y_{\text{body}}(t) = 0$.

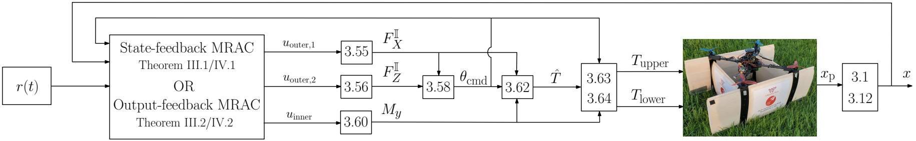


Control Framework

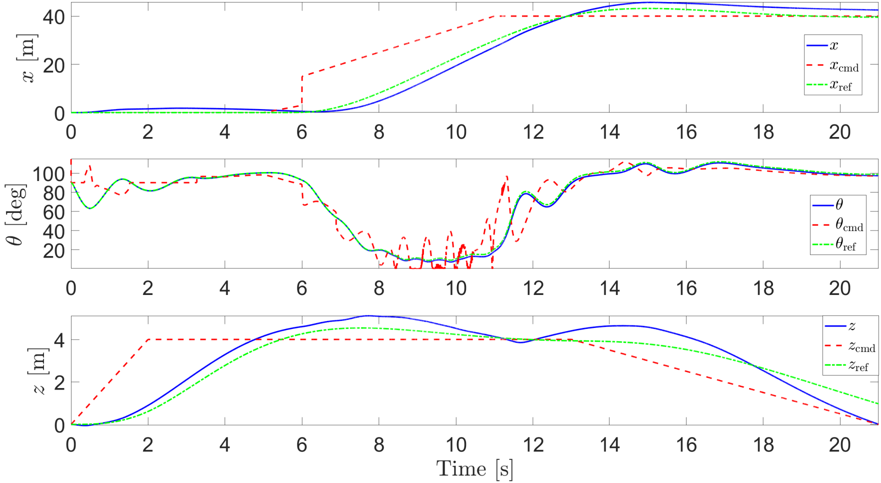
- MRAC for prescribed performance:
  - Constrain the trajectory tracking error;
  - Set rate of convergence of the error;
  - Compensate for uncertainties.
- Output tracking:
  - Guarantee uniform boundedness of the measured output to user-defined reference output.

Control strategy:

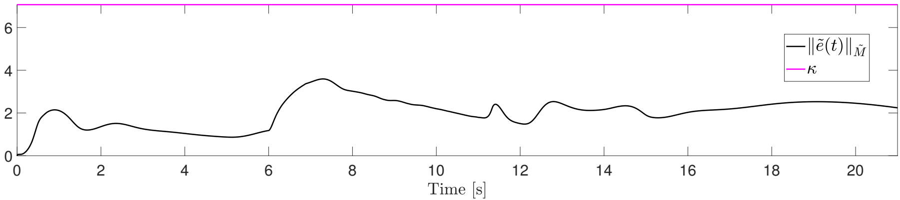
1. Define force to follow reference trajectory $\left[x_{A,\text{user}}^{\mathrm{T}}(t), z_{A,\text{user}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$ (Outer loop);
2. Identify orientation of desired thrust force $\theta_{\text{cmd}}(t)$;
3. Compute moment of thrust force to track required orientation (Inner loop).



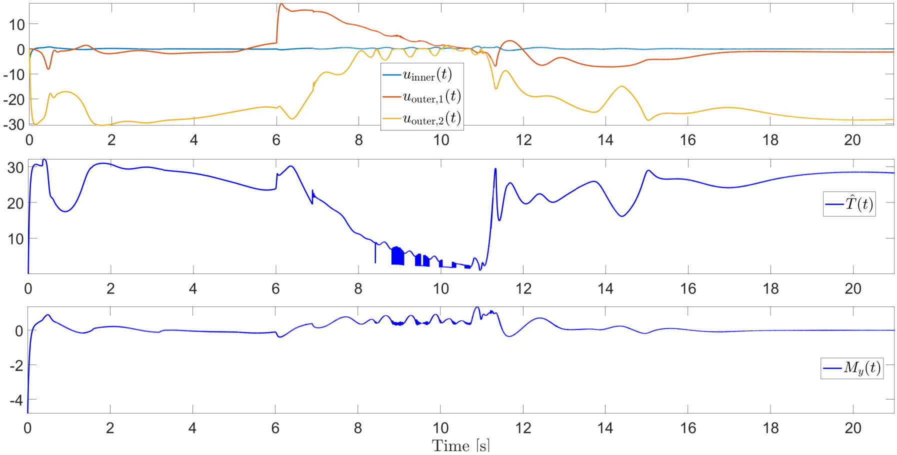
Output-feedback MRAC control results



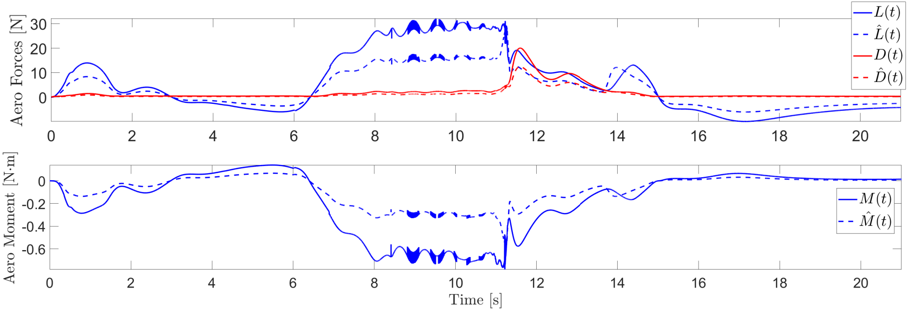
Output-feedback MRAC control results



Output-feedback MRAC control results

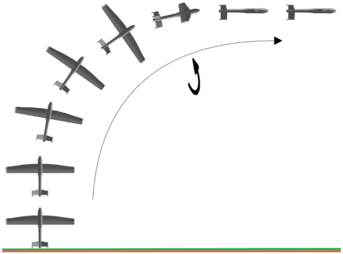


Output-feedback MRAC results



Tailsitter 6-DOF unified control system

- Designed a unified control system for tailsitters;
  - Does not distinguish among flight modes
  - Does not distinguish between lateral and longitudinal dynamics
- Two-layer MRAC framework;
  - Set rate of convergence of error
- Avoids unwinding by leveraging barrier functions;
- Novel method for deducing reference angular velocities.



Quadbiplane Preliminaries

- Four orthonormal reference frames;
  - Inertial: $\mathbb{I} = \{O; X, Y, Z\}$,
  - Body: $\mathbb{J} = \{C(\cdot); x_{\mathbb{J}}(\cdot), y_{\mathbb{J}}(\cdot), z_{\mathbb{J}}(\cdot)\}$,
  - Wind: $\mathbb{W} = \{C(\cdot); x_{\mathbb{W}}(\cdot), y_{\mathbb{W}}(\cdot), z_{\mathbb{W}}(\cdot)\}$.
  - Desired: $\mathbb{K} = \{r_{\mathrm{ref}}(\cdot); x_{\mathbb{K}}(\cdot), y_{\mathbb{K}}(\cdot), z_{\mathbb{K}}(\cdot)\}$.
- Euler parameters used to capture orientation: $q(t) \triangleq [q_1(t), q_{\mathrm{v}}^{\mathrm{T}}(t)]^{\mathrm{T}}$. The associated rotation matrix

$$R_{\mathbb{I}}^{\mathbb{J}}(q) \triangleq (2q_1^2 - 1)\mathbf{1}_3 + 2q_v q_v^{\mathrm{T}} + 2q_1 q_v^{\times}, \quad q \in \mathbb{H},$$

  where $\mathbb{H} \triangleq \{q \in \mathbb{R}^4 : \|q\| = 1\}$.
- Tracking error quaternion: $q_e(t) \triangleq q^{-1}(t) * q_{\mathrm{ref}}(t)$.
  - We want $\mathbb{J}$ to track a time-varying orthonormal reference frame $\mathbb{K} \triangleq \{C(\cdot), x_{\mathbb{K}}, y_{\mathbb{K}}, z_{\mathbb{K}}\}$, and avoid the unwinding phenomenon.

Rotational Kinematics

- The rotational kinematics

$$\dot{q}(t) = \frac{1}{2} J(q(t))\omega(t),$$

- $q_{\mathrm{ref}} : [t_0, \infty) \to \mathbb{H}$ is the differentiable reference quaternion capturing the attitude of $\mathbb{K}$ relative to $\mathbb{I}$,
- The command angular velocity

$$\omega_{\mathrm{cmd}}(q, q_e, \omega_{\mathrm{ref}}) \triangleq \omega_{\mathrm{ref}} + f_{\mathrm{constraint}}^{-1}(q_{e,1})\Gamma J^{\mathrm{T}}(q) q_{\mathrm{ref}},$$

where $\omega_{ref} \triangleq 2J^{\mathrm{T}}(q_{\mathrm{ref}}(t))\dot{q}_{\mathrm{ref}}(t)$, $\Gamma = \Gamma^{\mathrm{T}} > 0$, $f_{\mathrm{constraint}} : [-1, 1] \to \mathbb{R}$ be s.t. $f_{\mathrm{constraint}}(q_{e,1}) \geq 0$, $q_{e,1} \in [-1, 1]$, $f_{\mathrm{constraint}}(q_{e,1}) = 0$ i.f.f. $q_{e,1} = 0$, and both $f_{\mathrm{constraint}}(q_{e,1})$ and $\frac{\partial}{\partial q_{e,1}} f_{\mathrm{constraint}}(q_{e,1})$ are bounded for all $q_{e,1} \in [-1, 1]$.

Rotational Kinematics

- Consider the tracking error quaternion kinematics

$$\dot{q}_e(t) = \frac{1}{2}\left[J(q_e(t))\omega_{\mathrm{ref}}(t) - J_e(q_e(t))\omega(t)\right],$$
$$q_e(t_0) = q^{-1}(t_0) * q_{\mathrm{ref}}(t_0),\ t \geq t_0,$$

where $J_e(q_e) \triangleq \begin{bmatrix} -q_{e,\mathrm{v}}^{\mathrm{T}} \\ q_{e,1}\mathbf{1}_3 - q_{e,\mathrm{v}}^{\times} \end{bmatrix}$; note that $J_e(q_e) \neq J(q_e)$ for all $q_e \neq q_{\mathbb{U}}$.

Theorem

Consider $q_{\mathrm{ref}}(t)$, $t \geq t_0$, the corresponding reference angular velocity $\omega_{\mathrm{ref}}(t)$, and the tracking error quaternion kinematic equations above. If $\omega(t) = \omega_{\mathrm{cmd}}(q(t), q_e(t), \omega_{\mathrm{ref}}(t)), t \geq t_0$, then $\lim_{t\to\infty} q_e(t) = q_{\mathbb{U}}$ uniformly in $t_0 \geq 0$. Furthermore, if $q_{e,1}(t_0) > 0$, then $q_{e,1}(t) > 0$ for all $t \geq t_0$.

Equations of Motion

- Translational equations of motion

$$\ddot{r}_C^{\mathbb{I}}(t) = \frac{1}{m}\left[F_{\mathrm{T}}^{\mathbb{I}}(t) + mg\mathbf{e}_{3,3} + F_{\mathrm{A}}^{\mathbb{I}}(t, p(t), p_{\mathrm{dot}}(t))\right],$$

- The mass $m > 0$ is unknown, $F_{\mathrm{T}}(t) \triangleq u_1(t)\mathbf{e}_{1,3}$, and $F_{\mathrm{A}}^{\mathbb{I}}(\cdot)$ is the aerodynamic force.
- Rotational equations of motion

$$\dot{\omega}(t) = I^{-1}\left[M_{\mathrm{T}}(t) + M_{\mathrm{A}}(t, p(t), p_{\mathrm{dot}}(t)) - \omega^{\times}(t)I\omega(t)\right],$$

- $I = I^{\mathrm{T}} > 0$ is the matrix of inertia, $M_{\mathrm{T}} \triangleq [u_2(t), u_3(t), u_4(t)]^{\mathrm{T}}$ is the moment of the thrust force, and $M_{\mathrm{A}}(\cdot)$ is the aerodynamic moment.

Control Design

- Quadbiplanes are underactuated, cannot control x, y-position directly.
- The control system is separated into an outer and inner loop.
  - Outer loop computes:
    - Ideal thrust;
    - Reference attitude;
    - Reference angular rates to track user-defined trajectory.
  - Inner loop computes:
    - moment of the thrust force;
    - thrust allocation.

Outer Loop: Thrust Force Determination

- If both the direction and magnitude of the thrust force could be set arbitrarily

$$F_{\mathrm{T,ideal}} = u_{\mathrm{outer}} - R_{\mathbb{W}}^{\mathbb{J}}(\alpha,\beta)[\mathbf{e}_{1,3},\mathbf{e}_{2,3},0_3]\tilde{F}_{\mathrm{A}}^{\mathbb{W}}(t,p,p_{\mathrm{dot}}),$$

where $\tilde{F}_{\mathrm{A}}^{\mathbb{W}}$ captures an estimate of the aerodynamic force, and $u_{\mathrm{outer}} : [t_0,\infty) \to \mathbb{R}^3$ is the virtual control input is to be determined so that $\lim_{t\to\infty} \left\| \begin{bmatrix} r_C(t) \\ \dot{r}_C(t) \end{bmatrix} - \begin{bmatrix} r_{\mathrm{user}}(t) \\ \dot{r}_{\mathrm{user}}(t) \end{bmatrix} \right\| = 0$ uniformly in $t_0 \geq 0$.

- Thrust, $u_1(t)$ must be positive, the direction can be set by steering the vehicle's attitude.

$$F_{\mathrm{T,proj}}(t) \triangleq \left[\begin{bmatrix} H(F_{\mathrm{T,ideal}}^{\mathrm{T}}(t)\mathbf{e}_{1,3}) \\ 0_2 \end{bmatrix}, \mathbf{e}_{2,3}, \mathbf{e}_{3,3} \right] F_{\mathrm{T,ideal}}(t), \qquad t \geq t_0,$$

Outer Loop: Reference attitude

- The matrix which captures the least angular displacement between the current direction of the thrust and desired direction of the thrust

$$R_{\mathbb{J}}^{\mathbb{K}}(\tilde{q}_{\mathrm{ref}}(t)) \triangleq U(t)\tilde{\Sigma}(t)V^{\mathrm{T}}(t)$$

- $\tilde{\Sigma}(t) \triangleq \mathrm{diag}\{1, 1, \det(U(t)V^{\mathrm{T}}(t))\}$,
- $U, V : [t_0, \infty) \to \mathbb{R}^{3\times 3}$ denote the left and right singular vectors,
- $R_{\mathbb{J}}^{\mathbb{K}}(\tilde{q}_{\mathrm{ref}}(t))$ is the solution to the orthogonal Procrustes problem.
- We set $u_1(t) = H(\|F_{\mathrm{T,proj}}(t)\| - T_{\min})\|F_{\mathrm{T,proj}}(t)\|$ where $T_{\min} > 0$ is the minimum thrust force.

$$u_1(t)\mathbf{e}_{1,3}F_{\mathrm{T,proj}}^{\mathrm{T}}(t) = U(t)\Sigma(t)V^{\mathrm{T}}(t),$$

Outer Loop: Reference attitude

Theorem: Orthogonal Procrustes Problem

Consider the vector $u_1(t)\mathbf{e}_{1,3}$, $t \geq t_0$, where $u_1(t) = H(\|F_{\mathrm{T,proj}}(t)\| - T_{\min})\|F_{\mathrm{T,proj}}(t)\|$, the vector $F_{\mathrm{T,proj}}(t) = \left[\begin{bmatrix} H(F^{\mathrm{T}}_{\mathrm{T,ideal}}(t)\mathbf{e}_{1,3}) \\ 0_2 \end{bmatrix}, \mathbf{e}_{2,3}, \mathbf{e}_{3,3}\right] F_{\mathrm{T,ideal}}(t), t \geq t_0$, and $R^{\mathbb{K}}_{\mathbb{J}}(\tilde{q}_{\mathrm{ref}}(t)) \triangleq U(t)\tilde{\Sigma}(t)V^{\mathrm{T}}(t)$. It holds that

$$R^{\mathbb{K}}_{\mathbb{J}}(\tilde{q}_{\mathrm{ref}}(t)) = \mathrm{argmin}_{R \in SO(3)} \|u_1(t)R\mathbf{e}_{1,3} - F_{\mathrm{T,proj}}(t)\|, \qquad t \geq t_0,$$

Outer Loop: Reference angular rates

- $\tilde{q}_{\mathrm{ref}}(\cdot)$ which underlies $R_{\mathbb{K}}^{\mathbb{I}}(\tilde{q}_{\mathrm{ref}}) = U(t)\tilde{\Sigma}(t)V^{\mathrm{T}}(t)$ is not continuously differentiable, hence,

$$\omega_{\mathrm{ref}}(t) \triangleq 2J^{\mathrm{T}}(\tilde{q}_{\mathrm{ref}}(t))\dot{\tilde{q}}_{\mathrm{ref}}(t)$$

  cannot be computed directly.
- The spherical linear interpolation function, also known as geodesic curve, is defined as

$$\mathrm{slerp}(p,q,h) \triangleq p * \left(p^{-1} * q\right)^{h}, \qquad (p,q,h) \in \mathbb{H} \times \mathbb{H} \times [0,1].$$

- Given $p, q \in \mathbb{H}$, it holds that

$$\frac{\mathrm{d}}{\mathrm{d}h}\mathrm{slerp}(p,q,h) = \mathrm{slerp}(p,q,h) * \log(p^{-1} * q), \qquad h \in [0,1],$$
$$\frac{\mathrm{d}^2}{\mathrm{d}h^2}\mathrm{slerp}(p,q,h) = -\theta_{\mathrm{slerp}}^2\mathrm{slerp}(p,q,h),$$

Outer Loop: Reference angular rates

- Compute

$$
\begin{aligned}
q_{\text{ref}}(t) &= \lim_{\tau \to t} \operatorname{slerp}(q(t), \tilde{q}_{\text{ref}}(t), h(\tau)) \\
&= \tilde{q}_{\text{ref}}(t), \qquad t \geq t_0, \\
\dot{q}_{\text{ref}}(t) &= J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t}, \\
\ddot{q}_{\text{ref}}(t) &= J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}^2h(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=t} \\
&\qquad - \theta^2_{\text{slerp}}(t) q_{\text{ref}}(t) \left(\left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t} \right)^2,
\end{aligned}
$$

Outer Loop: Reference angular rates

- Problem: Is there a way to systematically choose the first & second derivatives of $h(\cdot)$?

$$
\begin{aligned}
\dot{q}_{\text{ref}}(t) &= J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t}, \\
\ddot{q}_{\text{ref}}(t) &= J(q_{\text{ref}}(t)) \left[\log(q^{-1}(t) * q_{\text{ref}}(t))\right]_{\text{v}} \left.\frac{\mathrm{d}^2h(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=t} \\
&\quad - \theta^2_{\text{slerp}}(t) q_{\text{ref}}(t) \left(\left.\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\right|_{\tau=t} \right)^2
\end{aligned}
$$

Outer Loop: Deducing $\frac{dh(\tau)}{d\tau}$ and $\frac{d^2h(\tau)}{d\tau^2}$

- Minimize

$$\sum_{i=1}^{n_t} \begin{bmatrix} \Omega_{\mathrm{ref},i} \\ \dot{\Omega}_{\mathrm{ref},i} \end{bmatrix}^{\mathrm{T}} \Theta \begin{bmatrix} \Omega_{\mathrm{ref},i} \\ \dot{\Omega}_{\mathrm{ref},i} \end{bmatrix}$$

s.t.

$$\begin{aligned}
H_{i+1} &= H_i + H_{i,\mathrm{dot}}\delta t + \frac{1}{2}H_{i,\mathrm{ddot}}\delta t^2, \quad i \in \{1, \ldots, n_t - 1\}, \\
H_{i+1,\mathrm{dot}} &= H_{i,\mathrm{dot}} + H_{i,\mathrm{ddot}}\delta t, \quad i \in \{1, \ldots, n_t - 1\}, \\
\Omega_{\mathrm{ref},i} &= 2[\log(q^{-1}(t) * q_{\mathrm{ref}}(t))]_{\mathrm{v}} H_{i,\mathrm{dot}}, \quad i \in \{1, \ldots, n_t\}, \\
\dot{\Omega}_{\mathrm{ref},i} &= 2[\log(q^{-1}(t) * q_{\mathrm{ref}}(t))]_{\mathrm{v}} H_{i,\mathrm{ddot}}, \quad i \in \{1, \ldots, n_t\}, \\
H_1 &= 0, \\
H_{n_t} &= 1, \\
H_i &\in [0, 1], \quad i \in \{2, \ldots, n_t - 1\},
\end{aligned}$$

Outer Loop: Deducing $\frac{dh(\tau)}{d\tau}$ and $\frac{d^2h(\tau)}{d\tau^2}$

- Minimize

$$z^{\mathrm{T}} R z,$$

s.t.

$$Cz = b,$$
$$Gz \leq d,$$

- Can be solved employing Newton-Raphson methods, or *fast* MPC.
- Without inequality constraints, an analytical solution exists if R is non-negative definite and C is full-row rank,

$$\begin{bmatrix} z^* \\ \lambda_1^* \end{bmatrix} = \begin{bmatrix} R & C^{\mathrm{T}} \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ b \end{bmatrix}.$$

MRAC System Design

- Let $\tilde{I} \in \mathbb{R}^{3\times 3}$ be a symmetric positive-definite matrix that captures an estimate of the matrix of inertia I,
- $\tilde{M}_{\rm A}(t,p,p_{\rm dot}) : [t_0,\infty) \times \mathbb{R}^3 \times \mathbb{H} \times \mathbb{R}^6 \to \mathbb{R}^3$ capture an estimate of the moment of the aerodynamic force

$$M_{\rm T}(t) = u_{\rm inner}(t) + \omega^{\times}(t)\hat{I}\omega(t) - \tilde{M}_{\rm A}(t,p(t),p_{\rm dot}(t)), \;\; t \geq t_0.$$

- The equations of motion reduce to

$$\dot{x}_C(t) = A_C x_C(t) + B_C \Lambda_C \left[u_{\rm outer}^{\mathbb{I}}(t) + \Theta_{\rm outer}^{\rm T} \Phi_{\rm outer}(t,p(t),p_{\rm dot}(t)) \right],$$

$$x_C(t_0) = x_{C,0}, \quad t \geq t_0,$$

$$\dot{\omega}(t) = I^{-1} \left[u_{\rm inner}(t) + \Theta_{\rm inner}^{\rm T} \Phi_{\rm inner}(t,p(t),p_{\rm dot}(t)) \right], \qquad \omega(t_0) = \omega_0,$$

where $x_C(t) \triangleq \left[\left(r_C^{\mathbb{I}}(t)\right)^{\rm T}, \left(\dot{r}_C^{\mathbb{I}}(t)\right)^{\rm T} \right]^{\rm T}$. MRAC form!

Aerodynamic Model

- We model the aerodynamic forces as

$$F_{\mathrm{A}}^{\mathbb{W}}(t,p,p_{\mathrm{dot}}) = \frac{1}{2}\rho\|\dot{r}_C\|S\begin{bmatrix} C_{\mathrm{D}_0} + k_{\mathrm{D}}C_{\mathrm{L}}^2(\alpha(t))\|\dot{r}_C\| \\ C_{\mathrm{S}_\beta}\beta(t)\|\dot{r}_C\| + \frac{b}{2}\left(C_{\mathrm{S}_p}\mathbf{e}_{1,3} + C_{\mathrm{S}_r}\mathbf{e}_{3,3}\right)^{\mathrm{T}}\left[R_{\mathbb{W}}^{\mathbb{J}}(\alpha(t),\beta(t))\right]^{\mathrm{T}}\omega \\ C_{\mathrm{L}}(\alpha(t))\|\dot{r}_C\| + \frac{c}{2}C_{\mathrm{L}_q}\mathbf{e}_{2,3}^{\mathrm{T}}\left[R_{\mathbb{W}}^{\mathbb{J}}(\alpha(t),\beta(t))\right]^{\mathrm{T}}\omega \end{bmatrix}.$$

- The aerodynamic lift coefficient is captured by

$$C_{\mathrm{L}}(\alpha) \triangleq [1-\sigma(\alpha)]\left[C_{\mathrm{L}_0} + C_{\mathrm{L}_\alpha}\alpha\right] + 2\sigma(\alpha)\mathrm{sign}(\alpha)\sin^2\alpha\cos\alpha, \qquad \alpha \in (-\pi,\pi],$$

$\mathrm{sign}(\cdot)$ denotes the signum function,

$$\sigma(\alpha) \triangleq \frac{1 + e^{-M_0(\alpha-\alpha_{\mathrm{stall}})} + e^{M_0(\alpha+\alpha_{\mathrm{stall}})}}{\left[1 + e^{-M_0(\alpha-\alpha_{\mathrm{stall}})}\right]\left[1 + e^{M_0(\alpha+\alpha_{\mathrm{stall}})}\right]}.$$

Translational error dynamics

- Introduce the reference model

$$\dot{x}_{C,\mathrm{ref}}(t) = A_{C,\mathrm{ref}} x_{C,\mathrm{ref}}(t) + B_C r_{\mathrm{ref}}(t), \quad x_{C,\mathrm{ref}}(t_0) = \begin{bmatrix} r^{\mathbb{I}}_{\mathrm{user}}(t_0) \\ \dot{r}^{\mathbb{I}}_{\mathrm{user}}(t_0) \end{bmatrix}, \quad t \geq t_0$$

- The trajectory tracking error dynamics is captured by

$$\dot{e}_C(t) = A_{C,\mathrm{ref}} e_C(t) + B_C \Lambda_C \left[u^{\mathbb{I}}_{\mathrm{outer}}(t) - K^{\mathrm{T}}_{\mathrm{outer}} \pi_{\mathrm{outer}}(t, p(t), p_{\mathrm{dot}}(t)) \right],$$

$$e_C(t_0) = x_C(t_0) - x_{C,\mathrm{ref}}(t_0),$$

where $K_{\mathrm{outer}} \triangleq \left[K^{\mathrm{T}}_{x_C}, m\mathbf{1}_3, \tilde{\Theta}^{\mathrm{T}}_{\mathrm{outer}} \right]^{\mathrm{T}}$.

- Introduce the reference model for the error's transient

$$\dot{e}_{C,\mathrm{tran}}(t) = A_{C,\mathrm{tran}} e_{C,\mathrm{tran}}(t), \quad e_{C,\mathrm{tran}}(t_0) = e_{C,0}, \quad t \geq t_0,$$

where $A_{C,\mathrm{tran}}$ is Hurwitz and s.t. $\mathrm{Re}(\lambda_{\max}(A_{c,\mathrm{tran}})) < \mathrm{Re}(\lambda_{\min}(A_{C,\mathrm{ref}}))$

Rotational error dynamics

- We introduce the system

$$\dot{x}_{\omega,\mathrm{ref}}(t) = -\zeta x_{\omega,\mathrm{ref}}(t), \qquad x_{\omega,\mathrm{ref}}(t_0) = \omega_0, \qquad t \geq t_0,$$

where $\zeta > 0$ is user-defined, and we note that

$$\dot{e}_\omega(t) = -\zeta e_\omega(t) + I^{-1}[u_{\mathrm{inner}}(t) - K_{\mathrm{inner}}^{\mathrm{T}} \pi_{\mathrm{inner}}(t, p(t), p_{\mathrm{dot}}(t))],$$
$$\Delta\omega(t_0) = 0,$$

where $e_\omega(t) \triangleq \Delta\omega(t) - x_{\omega,\mathrm{ref}}(t)$, and $\Delta\omega \triangleq \omega(t) - \omega_{\mathrm{cmd}}(t)$.

- Similarly,

$$\dot{x}_{\omega,\mathrm{tran}}(t) = -\zeta_{\mathrm{tran}} x_{\omega,\mathrm{tran}}(t), \quad x_{\omega,\mathrm{tran}}(t_0) = \omega_0 - x_{\omega,\mathrm{ref},0},$$

where $\zeta_{\mathrm{tran}} > \zeta$.

Feedback control laws

- The feedback control law for $u_{\text{outer}}(\cdot)$

$$\phi_{\text{outer}}(t, p, p_{\text{dot}}, \hat{K}_{\text{outer}}) \triangleq \hat{K}_{\text{outer}}^{\text{T}}(t)\pi_{\text{outer}}(t, p, p_{\text{dot}}) + \hat{K}_{\text{outer},g}^{\text{T}} e_{C,\text{tran}}(t) - K_{\text{PD,outer}}\left(x_C(t) - \left[r_{\text{user}}^{\text{T}}(t), \dot{r}_{\text{user}}^{\text{T}}(t)\right]^{\text{T}}\right),$$

  where $\pi_{\text{outer}}(t, p, p_{\text{dot}}) \triangleq \left[x_C^{\text{T}}, r_{\text{ref}}^{\text{T}}(t), -\tilde{\Phi}_{\text{outer}}^{\text{T}}(t, p, p_{\text{dot}})\right]^{\text{T}}$, $K_{\text{PD,outer}} \triangleq [K_{\text{P,outer}}, K_{\text{D,outer}}]$, $K_{\text{P,outer}}, K_{\text{D,outer}} \in \mathbb{R}^{3\times 3}$ are symmetric, positive-definite,
- The feedback control law for $u_{\text{inner}}(\cdot)$

$$\phi_{\text{inner}}(t, p, p_{\text{dot}}, q_{\text{ref}}, \hat{K}_{\text{inner}}) \triangleq \hat{K}_{\text{inner}}^{\text{T}}(t)\pi_{\text{inner}}(t, p, p_{\text{dot}}) + \hat{K}_{\text{inner},g}^{\text{T}} e_{\omega,\text{tran}}(t) + q_{e,1}^{-2} J^{\text{T}}(q) q_{\text{ref}},$$

  where $\pi_{\text{inner}}(t, p, p_{\text{dot}}) \triangleq \left[-\Phi_{\text{inner}}^{\text{T}}(t, p, p_{\text{dot}}), \dot{\omega}_{\text{cmd}}^{\text{T}}(q(t), q_e(t), \omega_{\text{ref}}(t)), \Delta\omega^{\text{T}}(t)\right]^{\text{T}}$.

Adaptive laws

- Two-layer MRAC

$$\dot{\hat{K}}_{\text{outer}}(t) = -\Gamma_{\text{outer}}\pi_{\text{outer}}(t, p(t), p_{\text{dot}}(t))\tilde{e}_C^{\mathrm{T}}(t)P_{\text{outer,tran}}B_C,$$
$$\hat{K}_{\text{outer}}(t_0) = \hat{K}_{\text{outer},0}, \quad t \geq t_0,$$

$$\dot{\hat{K}}_{\text{outer,g}}(t) = -\Gamma_{\text{outer},g}e_C(t)\tilde{e}_C^{\mathrm{T}}(t)P_{\text{outer,tran}}B_C,$$
$$\hat{K}_{\text{outer},g}(t_0) = \hat{K}_{\text{outer},0,g}, \quad t \geq t_0,$$

$\Gamma_{\text{outer}} \in \mathbb{R}^{(N_{\text{outer}}+9)\times(N_{\text{outer}}+9)}$, $P_{\text{outer,tran}} \in \mathbb{R}^{6\times 6}$ is symmetric, positive-definite, and such that

$$0_{6\times 6} = A_{C,\text{ref}}^{\mathrm{T}}P_{\text{outer,tran}} + P_{\text{outer,tran}}A_{C,\text{ref}} + Q_{\text{outer}},$$

$$\dot{\hat{K}}_{\text{inner}}(t) = -\Gamma_{\text{inner}}\pi_{\text{inner}}(t, p(t), p_{\text{dot}}(t))e_\omega^{\mathrm{T}}(t),$$
$$\hat{K}_{\text{inner}}(t_0) = \hat{K}_{\text{inner},0}, \quad t \geq t_0,$$

$\Gamma_{\text{inner}} \in \mathbb{R}^{(N_{\text{outer}}+3)\times(N_{\text{inner}}+3)}$ are symmetric, positive-definite, and user-defined.

Stability analysis

Theorem

If $u_{\text{outer}}(t) = \phi_{\text{outer}}(t, p(t), p_{\text{dot}}(t), \hat{K}_{\text{outer}}(t))$, $t \geq t_0$, and $u_{\text{inner}}(t) = \phi_{\text{inner}}(t, p(t), p_{\text{dot}}(t), \hat{K}_{\text{inner}}(t))$, then $\lim_{t\to\infty} \left\| x_C(t) - \begin{bmatrix} r_{\text{user}}(t) \\ \dot{r}_{\text{user}}(t) \end{bmatrix} \right\| = 0$ and $\lim_{t\to\infty} q_e(t) = q_{\mathbb{U}}$ uniformly in $t_0 > 0$. Furthermore, if $q_{e,1}(t_0) > 0$, then $q_{e,1}(t) > 0$ for all $t \geq t_0$.

Thrust force realization

- The thrust force each propeller must deliver (if $u_1(t) \geq T_{\min} > 0$)

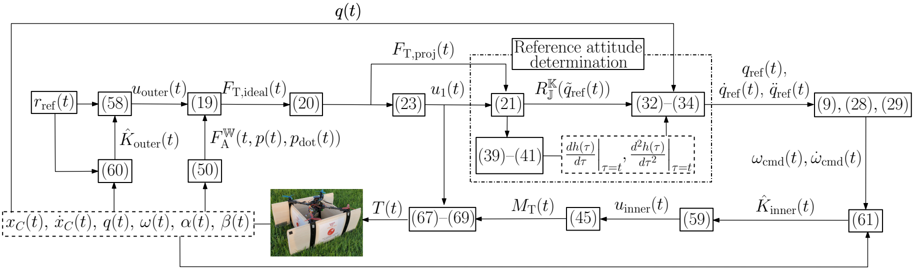
$$
\begin{aligned}
&\min_{T\in\mathbb{R}^4} \; T^{\mathrm{T}}(t)T(t), \qquad t \geq t_0,\\
&T(t) = \mathcal{M}^{-1}\begin{bmatrix} u_1(t) \\ M_{\mathrm{T}}(t) \end{bmatrix},\\
&T(t) \geq\geq \frac{T_{\min}}{4}[1,1,1,1]^{\mathrm{T}},
\end{aligned}
$$

where $T(t) \triangleq [T_{\mathrm{u,l}}(t), T_{\mathrm{u,r}}(t), T_{\mathrm{l,r}}(t), T_{\mathrm{l,l}}(t)]^{\mathrm{T}}$,

$$
\mathcal{M} \triangleq \begin{bmatrix} 1 & 1 & 1 & 1 \\ -c_{\mathrm{D}} & c_{\mathrm{D}} & -c_{\mathrm{D}} & c_{\mathrm{D}} \\ -l & -l & l & l \\ l & -l & -l & l \end{bmatrix},
$$

- $l > 0$: distance of the propellers from the UAV's longitudinal plane,
- $c_{\mathrm{D}} > 0$: propeller's *drag coefficient*,
- $\geq\geq$ denotes the component-wise inequality.

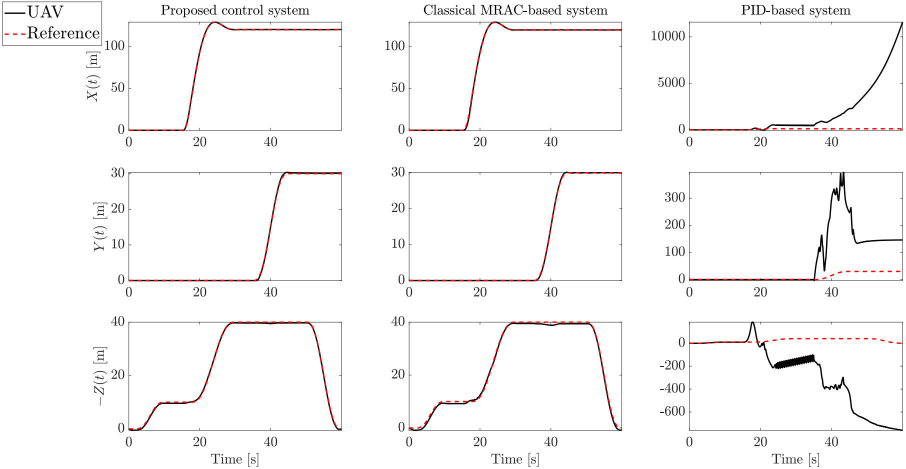
Control schematic



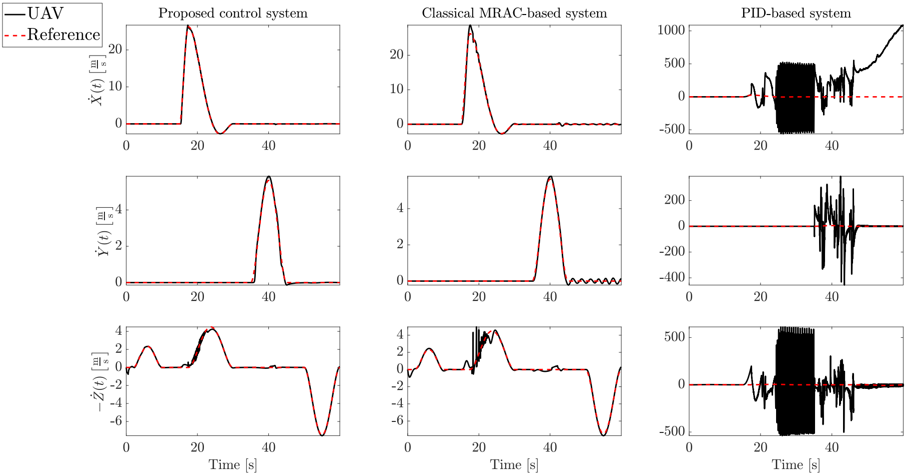
Numerical simulations: Setup

- Comparison with alternative control systems
  - Classical MRAC;
  - PID/PI, no unwinding constraints:
- Mission:
  1. Vertical takeoff 10m,
  2. Forward flight 140m,
  3. Vertical ascent to 40m altitude,
  4. Lateral flight 30m,
  5. Vertical landing.
- 40% uncertainty in all parameters.

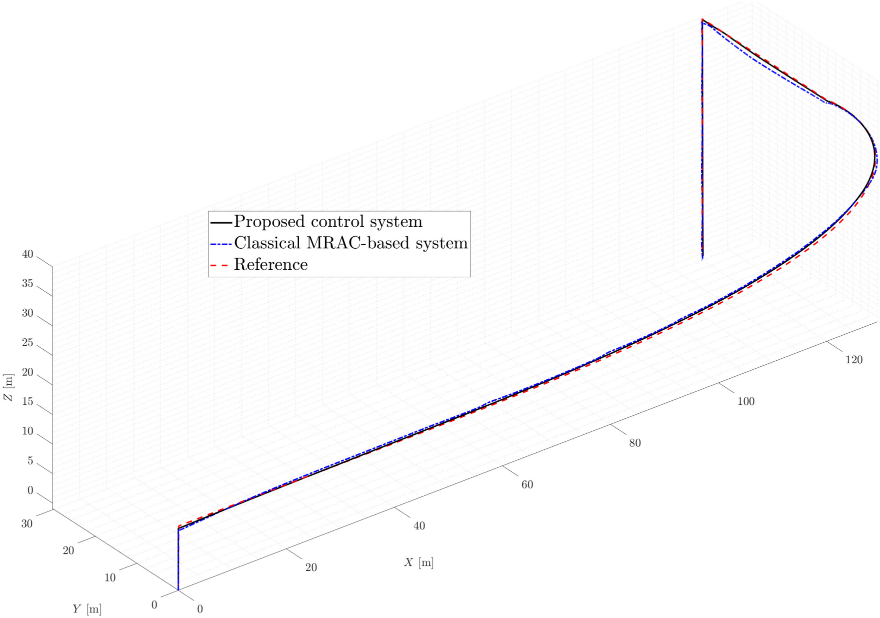
Numerical simulations: Results



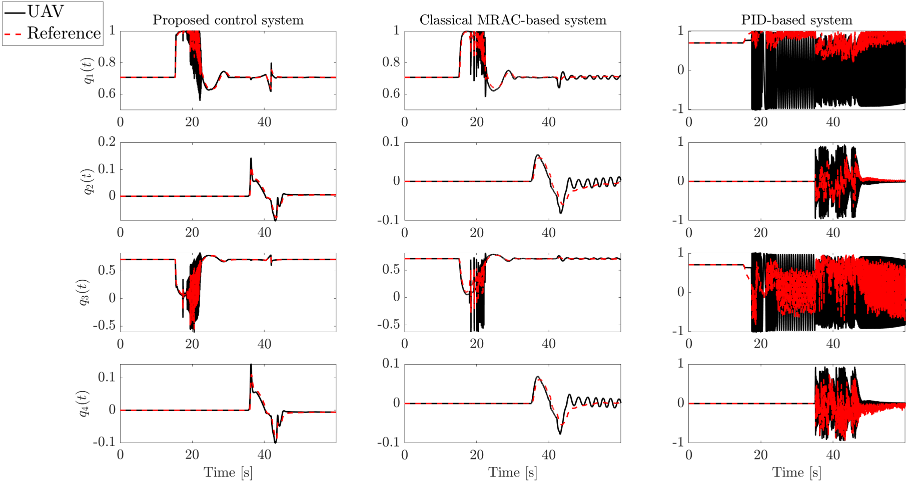
Numerical simulations: Results



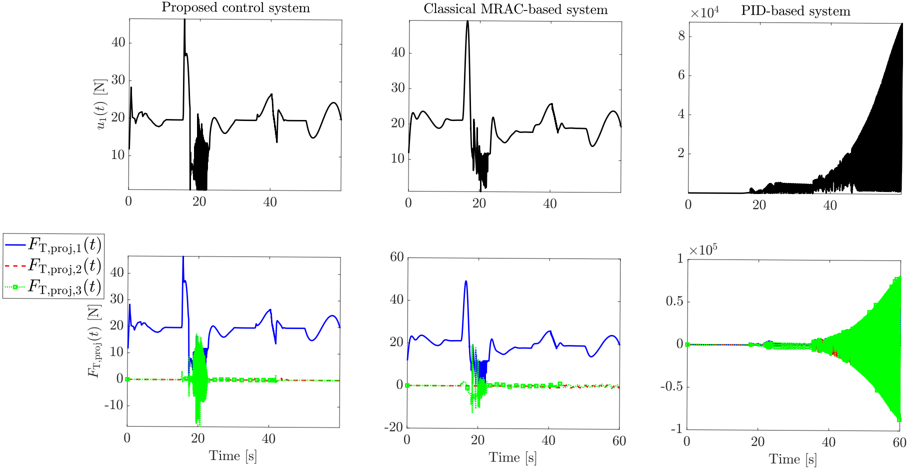
Numerical simulations: Results



Numerical simulations: Results



Numerical simulations: Results



Final remarks

- Design of feasible reference trajectories can be completed using nonlinear model predictive control;
- Unified approach may lead to a more complicated control design, future works involve using a switched control architecture;

Sponsors:

- NSF Grant no. 2137159
- U.S. DoN Grant no. N004212110004

J. Marshall and A. L'Afflitto. Two-Layer Model Reference Adaptive Control of a Tailsitter UAV with Unwinding Constraints, Aerospace Science & Technology - Submitted.